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Reconsideration of Dimensions and Curve Fitting Practice in Economics Ellaborating on Georgescu-Roegen's Economic Methodology

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Reconsideration of Dimensions and Curve Fitting Practice in Economics Ellaborating on Georgescu-Roegen's Economic Methodology[1]

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Abstract

This paper is to examine the proper use of dimensions and curve fitting practices elaborating on Georgescu-Roegen's economic methodology in relation to the three main concerns of his epistemological orientation. Section 2 introduces two critical issues in relation to dimensions and curve fitting practices in economics in view of Georgescu-Roegen's economic methodology. Section 3 deals with the logarithmic function (ln z) and shows that z must be a dimensionless pure number, otherwise it is nonsensical. Several unfortunate examples of this analytical error are presented including macroeconomic data analysis conducted by a representative figure in this field. Section 4 deals with the standard Cobb-Douglas function. It is shown that the operational meaning cannot be obtained for capital or labor within the Cobb-Douglas function. Section 4 also deals with economists' "curve fitting fetishism". Section 5 concludes this paper with several epistemological issues in relation to dimensions and curve fitting practices in economics.

Keywords: dimensions; logarithmic function; Nicholas Georgescu-Roegen; macroeconomics; Cobb-Douglas function; econometrics; curve fitting; transcendental production function

JEL Classification: B41, C01, E01

[1] This work represents a step forward respect to a previous work on this issue that can be found at http://www.h-economica.uab.es/wps/2010_01.pdf

1. Introduction

Nicholas Georgescu-Roegen was one of the first economists to rigorously investigate the crucial interplay between economic activity and the natural environment in the light of thermodynamics and evolutionary perspectives. According to Georgescu-Roegen, nature consists only of what can be perceived. Beyond this limited perception of nature, there are only hypothesized abstractions (Georgescu-Roegen 1976). It is a happy surprise for us to see that Douglass C. North, who has made a seminal contribution to the analysis of institutional changes, shares a similar opinion with Georgescu-Roegen on this point: "the world we have constructed and are trying to understand is a construction of the human mind. It has no independent existence outside the human mind" (North 2005, p.83). In our view, Georgescu-Roegen's ideas about the relation between nature and the human perception of nature led him to a particular epistemology concerned mainly with (1) how to establish a valid analytical representation of relations among the imperfectly perceived facts in the economic process; (2) whether or not the analytical representation is robust over time in view of evolutionary circumstances of the economic process; and (3) under what conditions the statistical procedure to check the validity of the analytical representation can be properly conducted.

In relation to the valid analytical representation of relations among facts, proper use of dimensions is a prerequisite to conduct any science. What we mean by dimensions here are the elementary units (such as mass, length, time, or money) referring to the definition of an external referent required for obtaining empirical data expressed as quantitative values assigned to the chosen proxy variables.[1] Yet, it seems that due attention has not been paid to proper use of dimensions in economics to be shown in this paper.

In relation to the two other issues, the robustness of the analytical representation and the appropriate use of statistical techniques, a critical evaluation of the curve fitting practice in economics is necessary. High speed computers are easily available to anybody at a low cost. Additionally, availability of statistical packages for the purpose of curve fitting has dramatically increased. Under these circumstances, easy-going so-called empirical works have become dominant and ubiquitous in economic science. Yet not many researchers have raised a serious doubt about the empirical validity of statistical procedures.[2]

The main purpose of this paper is to examine the proper use of dimensions and curve fitting practices elaborating on Georgescu-Roegen's economic methodology in relation to the three main concerns of his epistemological orientation. Section 2 briefly introduces two critical issues in relation to dimensions and curve fitting practices in economics in view of Georgescu-Roegen's economic methodology. Section 3 first deals with the logarithmic function (ln *z*) and then shows that *z* must be a dimensionless pure number, otherwise it is nonsensical. Several unfortunate examples of this analytical error are presented including macroeconomic data analysis conducted by a representative figure in this field. Section 4 first deals with the standard Cobb-Douglass function. It is shown that the operational meaning cannot be obtained for each term, capital or labor, within the Cobb-Douglas function. Section 4 also deals with economists' "curve fitting fetishism". We claim that it is essential to make a clear distinction between curve fitting over past observations and the development of a theoretical or empirical law that must be capable of fitting future observations. Section 5 concludes this paper with several epistemological issues in relation to dimensions and curve fitting practices in economics.

2. Georgescu-Roegen's critique of dimensions and curve fitting practice in economics revisited

The entire spectrum of Georgescu-Roegen's fertile and profound works covers every important aspect of economic science. In this section we touch upon only two issues he investigated: (1) a critical appraisal of proper use of dimensions and (2) a critical appraisal of the econometric approach and procedures to the economic process.

We believe that the vast majority of readers might wonder why dimensions matter in economics. Perhaps it is not well known that one of the first four papers published by Georgescu-Roegen in *Quarterly Journal of Economics* was concerned with the dimensions in relation to the marginal utility of money (Pigou et al. 1936). This paper has given a correct (and sober) verdict on a famous controversy between A. C. Pigou and Milton Friedman based on the following series where p_i stands for the price of commodity *i* and φ stands for utility.

$$T = \frac{p_x^2}{\varphi_{xx}} + \frac{p_y^2}{\varphi_{yy}} + \frac{p_z^2}{\varphi_{zz}} + \ldots.. \qquad (1)$$

Here *T* is not a pure number. Rather, it has the dimension $(\text{money})^2/(\text{utility})$. This is an important point, since for example, a change from US dollars to cents increases the numerical value of *T* by 10,000 times. Georgescu-Roegen states: "As there is no sense in speaking of a dimensional quantity as small or large, the difficulty of dimension arises at once when we pass from a mathematical constant to a quasi-constant. What do we mean, for instance, by *T* becoming very large as the number of commodities increases? First of all, how can we recognize whether *T* is large or small? By choosing appropriate units [dimensions] of measurement, *T* can be made to have any numerical value we please. We may try to avoid this question of dimensions by assuming the units are fixed once and for all. But this assumption does not help toward proving that the numerical value of *T* will increase indefinitely with the number of commodities" (Pigou et al. 1936, p. 535).

In another paper, "Mathematical Proofs of the Breakdown of Capitalism" in *Econometrica,* he has shown that "the Marxist scheme of expanded reproduction cannot be cast into a *mathematically* correct model" (Georgescu-Roegen 1960, p. 226). He has identified the purely analytical fallacy, which is one of our main points in this paper, in the Marxist formulation of expanded reproduction. In essence this concerns the issue of *the principle of dimensional homogeneity:* dimensionally different numbers cannot be summed up. One of the results that Georgescu-Roegen has proved concerning the dimensional homogeneity issue can be summarized in the following relation,

$$\bar{s} = l + v + k + \frac{dl}{dt} \quad (2)$$

where $\bar{s}$ is the surplus value, *l* the consumption of capitalists' households, *v* the increment of variable capital, *k* the increment of constant capital, and *t* time. Concerning the violation of the dimensional homogeneity in relation (2), Georgescu-Roegen states that "[a]s long as the letters in that formula stand for measurable material concepts and not for some Hegelian ideals, *l* and *dl/dt* cannot be added, any more than can total and average cost, for instance" (Georgescu-Roegen 1960, p.229) . Georgescu-Roegen has identified the arithmetical incongruity, i.e., the violation of dimensional homogeneity that reflects a neglected yet unfortunate aspect of Marxist economics.

Georgescu-Roegen has also provided a similar discussion on the same issue in relation to Marshall's constancy of marginal utility of money (Georgescu-Roegen, 1968).

On the other hand, Georgescu-Roegen's critique of the curve fitting practice in economics can be summarized as follows (e.g., Georgescu-Roegen 1952; 1966; 1971; 1976; 1979):

(i) Are the following crucial assumptions in econometrics acceptable?

The entire edifice of statistical theory rests on the general assumption that the relation between any sample produced by an *assumed* random mechanism and the parent population is "isomorphic" to each other. Most econometricians have assumed, implicitly as well as explicitly, that *all* economic data fulfill this isomorphism assumption and yet no justification other than mere verbalism has been offered in support of this position. However, in the social sciences, such as economics, it is perhaps impossible to point to the parent population correctly. This translates into the plausible proposition: a proof of the randomness of econometric data is impossible. In agronomy for instance, it is reasonable to assume that any group of observations is a random sample because we can experiment with the *same type* of fertilizer on as many plots selected at random as we please.

The most popular tests invoked in support of the reliability of an econometric model, the t-test, the F-test, and the z-test, all require that the sample be chosen at random from a *normal* population. Consequently, even if one would deal with data that can be safely regarded as constituting a random sample, before applying any of these tests one also needs to make sure that the parent population is normal. According to Georgescu-Roegen: "A number of doctoral candidates, who at my insistence have tested the normality of some of the data used in their dissertations, have all obtained decisively negative results" (Georgescu-Roegen 1976, p. 262). Georgescu-Roegen also states: "In this situation, to claim the validity of an econometric model on the basis of, say, the *F-test* is tantamount to claiming that a patient does not have cancer because his blood test for sugar has come out negative" (Georgescu-Roegen 1976, p. 262).

Since it is plausible that the parent population must be changing over time in terms of stochastic nature and its attributes, regarding time series data as a random sample is simply absurd.

(ii) Econometric practice for curve fitting is blind to changes associated with evolutionary economic process

Evolutionary factors play a substantial role and yet cannot be caught in an arithmomorphic (or mathematical) scheme. This point pertains to the confusion between discovering a quantitative law from a series of data and merely fitting a mathematical formula to the same data. The confusion thrives on the characteristic fluidity of the phenomenal domain of economics: almost any economic phenomenon is a *potential* element of change for almost any other such phenomenon. That is why we profess the highest esteem for general equilibrium theories. In this we are, no doubt, right. But the case of econometric models - which generally aim at formulating precise quantitative
macroeconomic laws - is quite different. Still worse, without the possibility of a controlled experiment, we can never discover the analytical law. Even more crucial is the absence of any concern for whether the formula obtained will also fit other observations. It is this concern that is responsible for the success natural scientists have with their formula. We find the distinction made by Faber and Proops (1998), in this regard, between *phenotypic evolution* (different realizations of potentialities of the systems, which are susceptible of prediction) and *genotypic evolution* (emergence of new institutions or techniques, which by definition are unpredictable; that is, new potentialities) rather interesting.

It must be remembered that without having chosen a model prior to the study of the statistical inference, the econometrician cannot solve any problem. On the other hand, the statistical inference will *confirm the choice of any model as far as the statistical test is positively confirmed in one way or another*. Under these circumstances it is ridiculous to see the practice, still widespread among economists and econometricians, , to transform the economic data and combine them in numberless ways until a satisfactory fit is obtained. The practice differs little from the more conspicuous form of pseudo-scientific endeavor. And with the increasing facilities of the computer utilization, the practice is likely to become predominant, at least by numbers.

3. The logarithmic function and dimensions: a fatal analytical fallacy

Even the layperson understand the meaning and implications of dimensional homogeneity emphasized above in Georgescu-Roegen's economc methodology: two numbers with different dimensions cannot be added, thus the sum "10 kg" plus "20 m^2" does not make any sense (Mayumi and Giampietro, 2010). A corollary of this principle is that it is meaningless to put a dimensional argument in the logarithmic function. Yet it is quite surprising to see that many economists violate this fundamental principle of arithmetic to be shown in this section. In particular, many economists put dimensional arguments in a logarithmic function. We present these observations with the hope that economists will orient future quantitative economic analysis toward more constructive ends without making shameful analytical errors into discussions on theoretical and empirical problems.

The logarithmic function belongs to a class of functions (i.e., the transcendental function) that also includes the exponential function and the trigonometric function. A transcendental function is a function that does not satisfy a polynomial equation. That is to say, a transcendental function is a function that "transcends" algebra in the sense that the function can never be represented in terms of a *finite* sequence of the algebraic operations of addition, subtraction, multiplication, division and root operations. Therefore, putting dimensional arguments in a transcendental function is an analytical error. However, since many practitioners in economics still put dimensional arguments in the logarithmic function, it is really instructive to directly show this analytical absurdity.

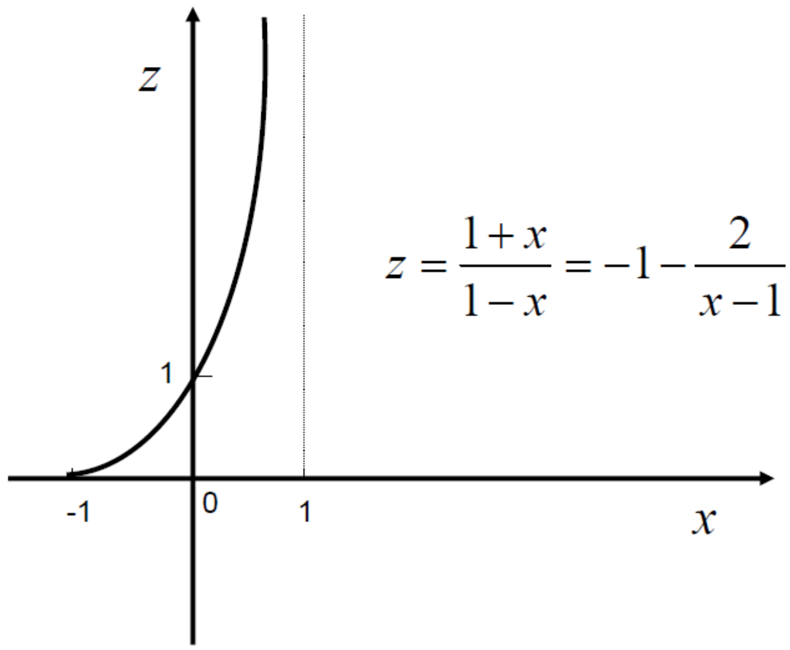


Figure 1. The monotonically increasing function z(x) for -1<x<1.

Let's start with the following expression[3],

$$\log_e(1+x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} +,,, + \frac{(-1)^{n-1}x^n}{n} +,,,, \quad (3)$$

where $-1 < x \le 1$ (4)

Replacing *x* by −*x* in relation (3) produces the following,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} -,,, - \frac{x^n}{n} +,,,, \quad (5).$$

Combining these two expressions (3) and (5) we have the following,

$$\ln\left(\frac{1+x}{1-x}\right) = 2(x + \frac{x^3}{3} + \frac{x^5}{5} +,,, + \frac{x^{2m-1}}{2m-1} +,,,) \quad (6)$$

Therefore, a unique value of *x* (-1<*x*<1) exists corresponding to *z* which is positive, as shown in Figure 1. Thus, for every positive real number *z*, we can safely define the logarithmic function as follows using the relation $x = \frac{z-1}{z+1}$:

$$\ln z = 2\{\left(\frac{z-1}{z+1}\right) + \frac{1}{3}\left(\frac{z-1}{z+1}\right)^3 + \frac{1}{5}\left(\frac{z-1}{z+1}\right)^5 +,,,+ \frac{1}{2m-1}\left(\frac{z-1}{z+1}\right)^{2m-1} +,,,\} \quad (7)$$

It is obvious that if the value of z is expressed in US$, this operation will create both "a square dollar" and "a cubic dollar", which are nonsensical, let alone "higher order dollars". Putting dollar values in the logarithmic function is analytically absurd as shown in Figure 2 and Figure 3.[4]

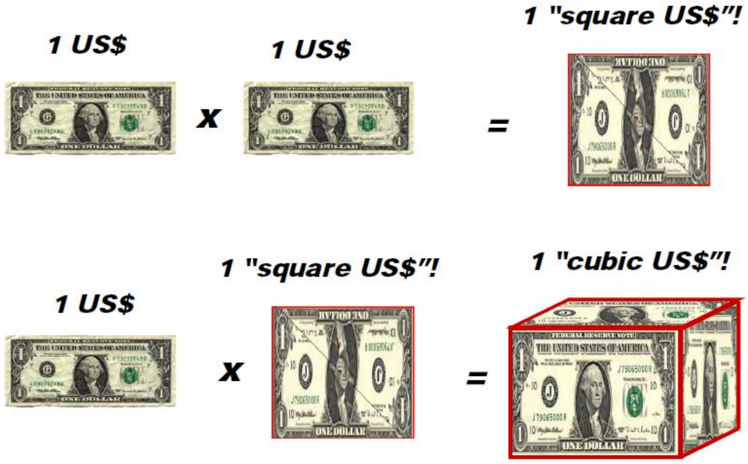
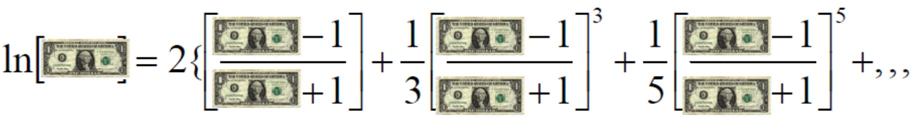


Figure 2. Fantastic Creation of Square and Cubic Dollars by Economists



Due to the violation of dimensional homogeneity, it is impossible to calculate 1 US dollar minus 1 or 1US dollar plus 1! We know that ln1=0 if 1 in ln is a pure number.

Figure 3. Putting 1 US Dollar in the Logarithmic Function

In April, 2009, we started a joint research project investigating general dimensional issues in empirical analysis. At the same time we *happened* to receive several papers dealing with the substitution among energy, capital and labor within the neoclassical production function framework from a young Italian friend of ours. These papers were full of logarithmic specifications of production functions. We identified several problems about the exponential and logarithmic functions within these papers. Then we tried to locate the dimensional errors, if any, in papers published in *Ecological Economics*. We found a few examples from the first two issues of *Ecological Economics* vol. 56 in 2006. The following unfortunate examples are from those papers as well as the papers that our Italian colleague supplied with us: Arrow et. al. 1961; Leontief, 1982; Morse 2006; Pastore et. al. 2000; Pyndick 1979; Richmond and Kaufmann 2006. Within this list we also cite our own error (Pastore et al. 2000). However, judging from our minimal "sampling procedure", we strongly believe that many other unfortunate examples are easily found in economic empirical analyses.[5]

It is very interesting to investigate when this unfortunate practice of putting dimensional arguments in the logarithmic function started. Our "educated guess" is that this analytical fallacy started with the publication of the classic article written by Christensen et al. (1973).[6] Pindyck's formulation cited above surely comes from the original formulation of transcendental logarithmic production and price frontiers investigated by Christensen et al. (1973).[7]

Consider their original formulation. They assumed that there are two outputs—consumption (*C)* and investment (*I)*—and two inputs—capital (*K)* and labor (*L)*. The corresponding prices are q_C, q_I, q_K, and q_L. They call *F* the production frontier in the following formulation,

$$\ln(F+1)=\alpha\ +\alpha_C\ln C+\alpha_I\ln I+\alpha_K\ln K+\alpha_L\ln L+\alpha_A\ln A+\ln C(\frac{1}{2}\beta_{CC}\ln C+$$

$$\beta_{CI}\ln I+\beta_{CK}\ln K+\beta_{CL}\ln L+\beta_{CA}\ln A)+\ln I(\frac{1}{2}\beta_{II}\ln I+\beta_{IK}\ln K+\beta_{IL}\ln L+\beta_{IA}\ln A)+$$

$$\ln K(\frac{1}{2}\beta_{KK}\ln K+\beta_{KL}\ln L+\beta_{KA}\ln A)+\ln L(\frac{1}{2}\beta_{LL}\ln L+\beta_{LA}\ln A)+\ln A(\frac{1}{2}\beta_{AA}\ln A)$$

(8), where, *according to the authors, A* is an index of technology.

It is not clear how to properly create this index. However, they use the price frontier as follows,

$$\ln(P+1)=\alpha_0+\alpha_C\ln q_C+\alpha_I\ln q_I+\alpha_K\ln q_K+\alpha_L\ln q_L+\alpha_A\ln A+\ln q_C(\frac{1}{2}\beta_{CC}\ln q_C+$$

$$\beta_{CI}\ln q_I+\beta_{CK}\ln q_K+\beta_{CL}\ln q_L+\beta_{CA}\ln A)+\ln q_I(\frac{1}{2}\beta_{II}\ln q_I+\beta_{IK}\ln q_K+\beta_{IL}\ln q_L+\beta_{IA}\ln A)+$$

$$\ln q_K(\frac{1}{2}\beta_{KK}\ln q_K+\beta_{KL}\ln q_L+\beta_{KA}\ln A)+\ln q_L(\frac{1}{2}\beta_{LL}\ln q_L+\beta_{LA}\ln A)+\ln A(\frac{1}{2}\beta_{AA}\ln A)$$

(9)

Since they clearly state that the "corresponding *prices* are q_C, q_I, q_K, q_L" (Christensen et al., 1973, p. 33, italics added), this specification cannot be used both in relation (8) and in relation (9).

Similarly macroeconomics often uses the logarithmic specification. Consider three papers of Lobert Lucas, Jr. that we happened to encounter during our writing of this paper, since he can be regarded an important representative of the macroeconomics field.

In the paper, "Making A Miracle" (Lucas 1993), *perhaps without any doubt* Allan D. Searle's result (1945), shown in Lucas' paper as Figure 1, is cited. According to Lucas, "Searle plotted man-hours vessel against number of vessels completed to date in that yard *on log-log paper* (Lucas 1993, pp. 259-260, italics added).

In another paper, "Macroeconomic Priorities" (Lucas 2003), Lucas states that "[u]sing annual U.S. data for the period 1947-2001, the standard deviation of *the log of real per capita consumption* about a linear trend is 0.0032" (Lucas 2003, p. 4, italics added).

In yet another paper, "Trade and the Diffusion of the Industrial Revolution" (Lucas 2009), he mentions that we "consider a world of one sector "AK" economies in which an economy's *GDP per capita* is proportional to its stock of human capital, knowledge capital, or whatever term you like" (Lucas 2009, p. 5). At this moment, we put aside the issue of measuring the amount of "knowledge capital" in concrete terms, which is itself a formidable task for any human beings. Lucas created four figures (Figure. 11, Figure 13, Figure 14 and Figure 15 in that paper) all of which have the same horizontal axis, *Log per capita GDP*. All these figures are nonsensical according to what has been said thus far.

At this moment we think that it is crucially important to note the following point. Suppose that an argument *a*, for instance per capita GDP, is represented in US dollars and we transform *a* into *b* represented in Japanese yen where *b=ea* and *e* is the exchange rate (yen/US dollar). Taking the natural logarithm on both sides (supposing this operation makes sense), we have

$$\ln b = \ln ea = \ln e + \ln a \qquad (10)$$

The readers must be convinced that the principle of dimensional homogeneity is totally violated, since the exchange rate *e* is transformed into $\ln e$ and added to $\ln a$.

Is it possible, therefore, for us to make an international comparison in Macroeconomics of per capita GDP if we transform per capita GDP into a logarithmic scale? Of course, not!

4. The Cobb-Douglas function and curve fitting fetishism in economics

We have examined how ridiculous it is to put dimensional arguments into the logarithmic function based on the dimensional homogeneity. However, we should also note that there are cases where certain types of algebraic operations on dimensional arguments become meaningless, as already shown in Figure 2. For the same reason we also examine whether or not each term represented in the Cobb-Douglas function has an operational meaning without any analytical fallacy like those we have identified in the case of the transcendental function, in particular the logarithmic function, before thoroughly discussing the curve fitting practice in economics.

We start with the standard Cobb-Douglas function as follows,

$$Y = AK^{\alpha}L^{1-\alpha} \qquad (11)$$

Suppose that *K, L*, and *Y* are represented in terms of the US dollar. Since $\alpha+(1-\alpha)=1$, the dimension of the left-hand side, the US dollar, is compatible with that of *the right-hand side as a whole* if A is a dimensionless pure number.

However, each term on the right-hand side, i.e., K^{α} and $L^{1-\alpha}$, does not make any sense unless $\alpha=0$ or 1. Suppose $\alpha=1/2$, is there any operational meaning of $\sqrt{100USdollar}$, for example?

Thus we are at a loss to understand the true reason why the Cobb-Douglas specification is often used in economic science. However, in fairness to Cobb and Douglas, the following fact must be emphasized. When we carefully read Cobb and Douglas' important classic paper (1928), one remains awed by their meticulous attitude. They devoted almost half of their paper to the task of how to create *the indices for capital and labor, not the prices*. They were also very careful about avoiding the generation of pseudo measures with the inconsistent ranking order of capital and labor indices.

In relation to curve fitting practices in economics, Georgescu-Roegen once aptly remarked (Georgescu-Roegen 1966, p. 277, italics added), "econometricians seem to ignore the fact that a better fit obtained by *adding a new variable* does not mean at all that the formula is also a better law. For a formula to represent a law it is not sufficient that it should fit well the available observations: the acid test is the fit for all other observations". The present situation for econometric analyses seems to have greatly worsened due to the increasing computational power of computers and programming techniques.

In mathematics there is a famous theorem called the Weierstrass Approximation Theorem: a real-valued continuous function can be approximated *uniformly* over a given domain by a polynomial (e.g., Randolph 1968). The uniform convergence means that for any given positive number ε (however small it may be) it is possible to create an approximate polynomial such that the absolute value of the distance (the norm) between the real-valued continuous function and the approximate polynomial can be less ε for a given domain.

For illustrational purposes we construct a polynomial series (the Bernstein polynomial) that uniformly converges to a continuous function *f*(*x*). The *n*th Bernstein polynomial for *f*(*x*) is constructed as follows,

$$B_n(x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right) {}_nC_k x^k (1-x)^{n-k} \qquad (12),$$

where

$$ {}_nC_k=\frac{n!}{k!(n-k)!} \qquad (13).$$

Suppose the following continuous function,

$$f(x)=\left(1+\frac{x}{8}\right)e^x \sin \pi x-\cos \pi x \qquad (14) \text{ where } x\in[0,1].$$

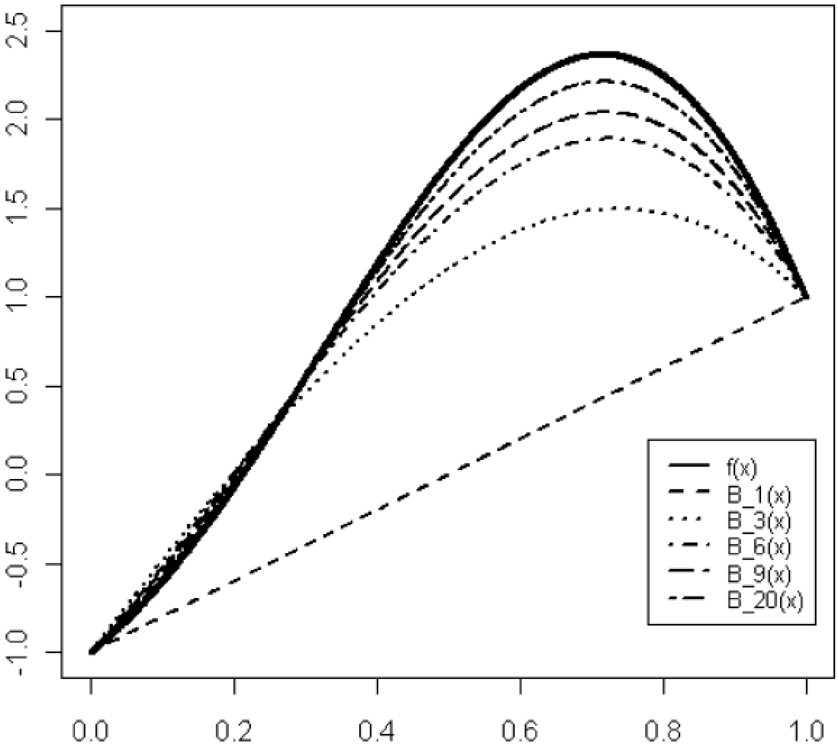


Figure 4. An Illustration of Weierstrass Approximation Theorem

Figure 4 shows a uniform convergence of $B_i(x)$ into $f(x)$. Raising the power of polynomials corresponds to Georgescu-Roegen's sense of adding new variables (or adding new parameters) in the analytical representation.

So, it is rather easy to have a polynomial approximation that can fit perfectly well to past data using computer programming. However, the situation facing economists is much more formidable. The "true function" $f(x)$ cannot be known in advance, especially if we seriously consider the evolutionary nature of the economic process!

The resulting curve fitting is a series of approximations that is supposed to be a real "law". Unfortunately $f(x)$ itself is simply *a formal representation of the perceived behavior of a system created by a modeler*. Therefore, this formal representation is based on: (1) the relevant system narrative adopted by the modeler; and (2) the data observed in the system and based on the perception of the modeler.

At this moment perhaps the vast majority of readers of this journal might argue that polynomials do not cover many functions that can be conceived in economic analysis. So it is better to explain without getting into mathematical technicalities why we consider the Weierstrass approximation theorem here. In mathematics there is a class of functions called measurable functions. Measurable functions cover almost any function used in econometrics. For this class of function there is a theorem (Lusin theorem, see e.g., Randolph 1968) that essentially states: for any measurable function there exists a continuous function over *almost everywhere* within the closed domain of the measurable function. That is to say, we can construct a continuous function that is almost identical to the original measurable function and the domains for both functions (the constructed continuous function and the original measurable function) are also almost identical for practical purposes of econometrics. Furthermore, polynomials are *dense* in the functional space of continuous functions due to the Werstrass approximation theorem, and we can approximate any conceivable function that is practically used in economics by polynomials as accurately as possible.[8]

5. Conclusion: the economic process and the true source of the limits of analytical representations

Concerning the issue of dimensions we have shown that it is an analytical fallacy to put the dimensional arguments in logarithmic functions and the meaningless variables in Cobb-Douglas functions.

When addressing the dimensions issue in relation to curve fitting practices in economics, there is an important epistemological problem. This problem regards the representation of the production process in quantitative terms. Neoclassical production functions, whether for individual firms or the aggregate economy, usually assume that any factor can always be substituted for any other factor. The implication of this assumption is that an increase in the input of any factor always yields an increase in output. For neoclassical economists any factor is a jelly-like substance, so that production is carried out everywhere in the input-output space. Such a space is assumed in the classic paper by H. S. Houthakker who formally derived the Cobb-Douglas production function based on the generalized Pareto distribution (Houthakker 1955). As S. Islam aptly showed, the second law of thermodynamics excludes the possibility of obtaining production isoquants of the Cobb-Douglass type (Islam, 1985). However, there is more to it. Those neoclassical economists adopting the substitution assumption have not paid due attention to the essential distinction between flows and funds in the material production process (Georgescu-Roegen 1971). This distinction leads to the heart of the issue which is the length of time horizon. It is the pre-analytical selection of a time horizon for the analysis, a descriptive domain associated with the choice of a given time scale, that defines what is produced by an economy. On a short time horizon one can decide to focus the analysis on the production of goods and services (performing an analysis of the flows). On a longer time horizon, when accounting for economic sustainability, one can decide to focus the analysis on the very processes required to produce and consume goods and services by performing an analysis of the reproduction and expansion of the funds. These two different types of analysis will provide different conclusions to the modeler and would require a different selection of models, variables and parameters. Neglecting the distinction between funds and flows (and neglecting the need of representing their production and reproduction using different attributes and models referring to different time scales) results in a systematic indifference to the biophysical foundation of economic activities. It is not surprising then that the curve fitting practice typical of aggregated production functions prevails.

Any actual material production process is limited in the sense that within a given factory process we cannot always compensate a decrease in output due to a decrease in a fund element (e.g. capital) by an increase in a flow input (e.g. natural resources). Hence, the representation of isoquants, the concept of elasticity of substitution, and the time derivative of a function by technological improvements, lose any operational and empirical meaning (Mayumi et al. 1998). All these concepts are found in the neoclassical theory of production. However, Georescu-Roegen noticed a much more serious "analytical and conceptual fallacy" within the neoclassical treatment of the development process: "It is high time, I believe, for us to recognize that the essence of development consists of the organizational and flexible power to create new processes rather than the power to produce commodities by materially crystallized plants" (Georgescu-Roegen 1971: 275). This power is termed as the Π-sector by Georgescu-Roegen (1971): "an economy can "take off" when and only when it has succeeded in developing a Π-sector". This issue of the Π-sector is related to the question of what is produced by the economic process. Some of those studying the functioning of socioeconomic processes seem to be confused as to what is actually produced by the economic process. According to Georgescu-Roegen, the economic process does not produce goods and services alone, but rather it produces a "reproducible system", via an integrated process of production and consumption of goods and services. When considering the whole socioeconomic system, it is the integrated action of the productive economic sector and the sector of final consumption which have to be considered. Using Georgescu-Roegen's terminology, the economic process has the goal of reproducing and expanding the various fund elements defined simultaneously across different levels and scales. It accomplishes this task by using disposable flows. Therefore, we can conclude that an economy not only produces goods and services*, but more importantly, produces the processes required for producing and consuming goods and services* (Giampietro and Mayumi, 2009; Mayumi 2009). This neglected aspect of the economic process in conventional economics is the true reason why curve fitting, based on dynamical system models and past data, results in continuous and inevitable failures to predict the future. At this moment, we should appreciate Marshall's description of what economics is about: "regarded as a branch of general history [economics] may aim at helping us to understand what has been the institutional framework of society at the several periods, what has been the constitution of the various social classes and their relation to one another" : it may "ask what has been the material basis of social existence; how have the necessities and conveniences of life been produced; by what organization has labour been provided and directed; how have the commodities thus produced been distributed; what have been the institutions resting on hits direction and distribution"; and so on" (Marshall 1920, p. 639).

Concerning the deficiency of dynamical systems analysis, Georgescu-Roegen had a serious concern with the abuse of mathematics. Georgescu-Roegen states: "Some aspects of [human society's] functioning lend themselves perfectly the mathematical analysis. Yet, when we come to the problem of its *evolution*, of its mutation into another form, mathematics proves to be too rigid and hence too simple a tool for handling it" (Georgescu-Roegen 1960, p. 243). In order to reinforce his arguments, it should be noted that even in natural sciences the severe limitations of mathematics are recognized by the authorities of this field. To wit: "even though the physicist's most dreadful weapon, mathematical deduction, would hardly be utilized. The reason for this was rather it was much too involved to be fully accessible to mathematics" (Schrödinger 1967, p. 3) and it "is the mathematics made by us which is imperfect and not our knowledge of nature" (Bridgman 1960. p. 62).

Concluding this overview of the epistemological challenges faced by those willing to generate a quantitative representation of the economic process, we can say that the validation of any dynamical system model can be assured only if both "the knowledge and the definition of the modeler" and the "observed system in the model" remain stable during the given time horizon. Put another way, the model remains valid only if the selected representation will not become either semantically (*phenotypic evolution*) or syntactically (*genotypic evolution*) obsolete over time (Ramos-Martin, 2003). Unfortunately, experience tells us that when dealing with the long-term historical analysis, these two conditions are never respected. For example, every time econometric models failed to predict energy demand, econometricians found a ready, yet self-defeating, excuse: "history has changed the parameters" (Georgescu-Roegen 1976). Georgescu-Roegen notes that if "history is so cunning, why persist in predicting it? What quantitative economics needs, above all, are economists such as Simon Kuznets, who would know how to pick out a small number of relevant variables, instead of relying upon the computer to juggle with scores of variables and thus losing all mental [introspective] contact with the dialectical nature of economic phenomena" (Georgescu-Roegen 1976).

The epistemological challenge associated with evolving systems is due to the mismatch between these two facts: (1) the information space used by any formal system of inference (mathematical model) must be closed, finite and discrete, otherwise it would not be possible to run such a model in finite time; (2) the information space for describing any evolving system is open and always expanding (Giampietro et al. EOLSS). By "information space" we mean the formal representation of the evolving system expressed in terms of the epistemological categories required to characterize its behavior. This implies that no matter how good a given model is, the simulated behavior always depends on the validity of the initial choice of typologies used in the representation. Unfortunately for modelers, individual realizations belonging to given typologies tend to evolve in time, "becoming" something else (Prigogine 1978). Thus, the validity of any model of an evolving system is bound to expire due to two plausible reasons:

(i) semantic obsolescence - the set of relevant attributes for the observed system must change in time, since the concerns justifying the model will naturally evolve with the advancement of knowledge. Thus, the qualities monitored and the priority given to various criteria of performance, will sooner or later cease to reflect the modeler's perception of relevance to the goals and problem structure (e.g. outdating of the narratives of neoclassical economics theory).

(ii) syntactic obsolescence - the set of relevant attributes for the observed system remains the same for the concerned modeler, but the model can no longer provide an accurate prediction of the values taken by key indicators, since the observed system has become something else (outdating of the validity of the curve fitting parameters). The model is no longer able to simulate the movements of the system within its original state space.

Judging from what we have presented in this paper, it is very difficult to accept the following statement advanced by Lucas: "Macroeconomics was born as a distinct field in the 1940s, as a part of the intellectual response to the Great Depression. The term then referred to the body of knowledge and expertise that we hoped would prevent the recurrence of that economic disaster. My thesis in this lecture is that macroeconomics in this original sense has succeeded: Its central problem of depression-prevention has been solved, for all practical purposes, and has in fact been solved for many decades" (Lucas 2003, p.1).

Concerning the difficulty in obtaining effective backward and forward feedbacks for controlling economic changes and avoiding catastrophic events, we conclude this paper with the following statements by Douglass C. North and Herbert A. Simon:

"Individuals act on incomplete information and with subjectively derived models that are frequently *erroneous*: the information feedback is typically insufficient to correct these subjective models" (North 1990, p. 16).

"In a world of uncertainty, no one knows the correct answer to the problems we confront and no one therefore can, in effect, maximize profits". (North, 1990 pag. 81)

"However, forming expectations to deal with uncertainty creates its own problems. Feedforward can have unfortunate destabilizing effects, for a system can overreact to its predictions and go into unstable oscillations. Feedforward in markets can become especially destabilizing, when each actor tries to anticipate the actions of the others (and hence their expectations)" (Simon 1996, p. 36).

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Notes

1. It should be noted that dimensions in mathematics such as the Hausdorff dimension or fractal dimensions (e.g., Hurewicz and Wallman, 1948; Edgar, 1990) have nothing to do with "dimensions" as discussed in this paper.

2. For the interested reader on one well known controversy, see (Hoover and Siegler 2008; McCloskey and Ziliak 1996).

3. Relation (3) can be obtained from the following expression:

$$\ln(1+x) = \int_0^x \frac{1}{1+u} du = \int_0^x (1-u+u^2-u^3+,,,+(-1)^{n-1}u^{n-1}+(-1)^n \frac{u^n}{1+u}) du .$$

4. In an interesting paper that properly criticizes the dimensional problems treated in neoclassical economics Barnett has made the same analytical fallacy, putting cm in the logarithmic function (Barnett 2004, p. 104). In physics and other natural science fields researchers often use the logarithmic function as if the normalization is already accomplished. In the case of Barnett's example, *it is very likely that* Barnett forgets the fact that distance is represented in cgs system, where 1

cm is used as a unit length. That means the number 14 is not 14 cm, but just a pure number 14, so that we can take logarithm without any problem.

5. To be fair to other economists, we have to acknowledge another error of ours to put dimensional arguments in the trigonometric function (Ramos-Martin et al. 2007).

6. However, this analytical fallacy might have been started much earlier judging from the publication by Allan D. Searle (1945) to be mentioned later in relation to Robert Lucas's analysis.

7. Since the empirical and theoretical studies in economics often adopt the logarithmic specification of the production and cost function, we derive a procedure or an algorithm, concerned with the given data set, by which we have examined whether or not a particular logarithmic specification is superior to the usual regression specification in terms of the least square norm and given a algorithm to be able to judge which specification is superior *only for the purpose of curve fitting* (Mayumi and Giampietro 2010). Needless to say, all the arguments in the data set are positive dimensionless pure numbers when they are put in the logarithmic specification.

8. Of course there exist very abnormally behaved functions within the measurable function, e.g., the function equal to 0 at all irrationals and 1 otherwise is measurable but discontinuous everywhere. This function (Dirichlet function) is represented as $f(x) = \lim_{m\to\infty} \lim_{n\to\infty} \{\cos(m!\pi x)\}^{2n}$ (e.g, Hausdorff 1937, p. 287). Dirichlet function is not continuous at a single point : it is nowhere continuous. The function is not integrable (in the sense of Riemann) over any small interval, but it is integrable in the sense of Lebesgue and its value of Lebesgue integration is zero!

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